Variations of the Componium

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Abstract

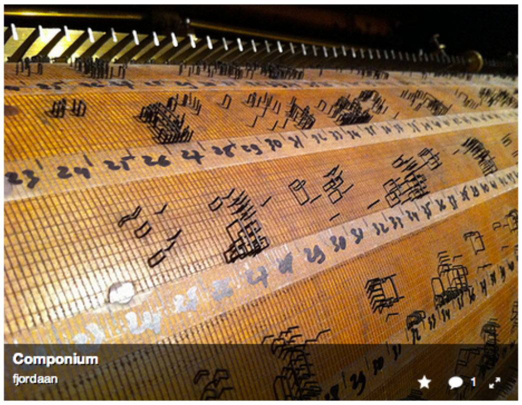
An analysis of the number of variations that can be produced by the Componium, Winkel's 1821 mechanical musical instrument. The componium was capable of a great number of variations using an aleatoric composition algorithm implemented via clockwork. This paper challenges an oft-repeated calculation first provided by Mahillon in 1880.

Introduction

The Componium, invented by Dietrich Nicholas Winkel and completed in 1821, is a remarkable early orchestrion, or mechanical orchestra. Like other orchestrions of the period, such as Maelzel's Panharmonicon which preceded it, it employed multiple ranks of organ pipes as well as a few percussion instruments. Only one Componium was ever built, and it can still be seen at the Museum of Musical Instruments (MIM) in Brussels, where it has resided since its acquisition in 1879. Currently unplayable, the instrument has been restored a handful of times (notably by famous magician Robert-Houdin in 1831, and by cellist Auguste Tolbecque in 1876). It was Tolbecque's collection that was acquired by the MIM, where the instrument was last restored and recorded in the 1960s [1].

The fascinating feature that distinguishes the Componium from other orchestrions was its ability to "improvise" or "extemporize" music as it played. The instrument was designed to play a piece of music of up to 80 measures in length. *

* A surviving set of barrels examined by John van Tiggelen employs only 76 measures, the remaining 4 measures are silent. We'll assume a full 80-measure piece for the remainder of this paper, since the instrument is capable of it.



There are two barrels mounted on the instrument. Upon the barrels are pinned 8 versions or variations of an 80-measure piece of music. The pieces are sub-divided into two-measure phrases, and the transcription alternates between the two barrels, each of which contains two measures of pins, followed by two measures of silence. These measures of silence are visible in the illustration as the gaps which extend across the barrel. Both barrels rotate simultaneously. As the top barrel plays two measures, the bottom barrel is silent. After two measures, the bottom barrel plays while the top barrel is silent. The two barrels alternate seamlessly in this way throughout the entire piece. When a particular barrel is playing, the silent barrel may slide horizontally, to activate one of the other seven variations. This sliding is governed by a randomizing mechanism similar to a roulette wheel. The end result is a piece of music which is coherent, yet highly variable.

The Componium was notable for being able to play a massive number of variations without repetition. When the instrument was first acquired by the MIM, its first curator, Victor-Charles Mahillon described it in the first volume of his catalogue, published in 1880. Mahillon provided some impressively large numbers. He wrote that the Componium could play 14,513,461,557,741,527,824 variations. He calculated that if each lasted five minutes, 138 trillion years would elapse before all possible combinations had been exhausted [2].

This quote has been repeated and paraphrased, in whole or in part, unchallenged, in a number of classic reference works which describe the Componium, including Lyr/Chapius [3] and Buchner [4], as well as the most thorough reference to date, a 1989 thesis by van Tiggelen [1].

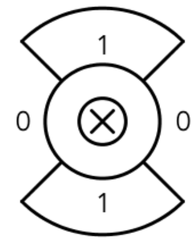
Unfortunately, Mahillon's estimate was miscalculated and is grossly inflated. The actual number, while still impressive, is significantly smaller.

How the Componium Works

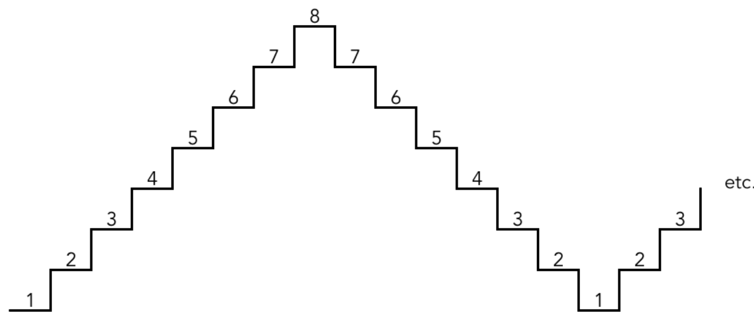
When I first attempted to work out the maximum number of variations the Componium can produce, I assumed that the aleatoric mechanism chose one of 8 variations each time it was employed (like an 8-sided roulette wheel). This would result in 8^{40} variations or 1,329,227,995,784,915,872,903,807,060,280,344,576 – a number dizzyingly larger than Mahillon's estimate.

I was initially mistaken, however, about how the instrument works, due to the lack of details in the sources I was reading. Both Mahillon and van Tiggelen have described the working of the instrument in sufficient detail.

The roulette-like wheel (one for each barrel) that introduces randomness does not select from 8 variations. It only serves to make a two-state decision, like the flip of a coin. This decision indicates whether the barrel stays on the same variation or advances to the next variation.

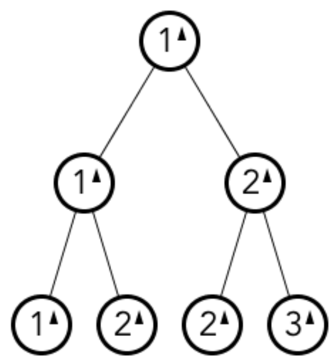


The variation number is controlled by a more predictable staircase gear (or "snail"), one for each barrel, which follows a 14-step pattern as shown to the right. In this way, the barrel only needs to slide a short distance in the scant few seconds of silence during which it has the opportunity to change variations. If the barrel is currently on variation 1 it will either stay on 1, or it will slide to variation 2. If the barrel is currently on variation 2, it may stay on 2, or depending on the position of the staircase gear (ascending or descending) it will slide to 1, or slide to 3.

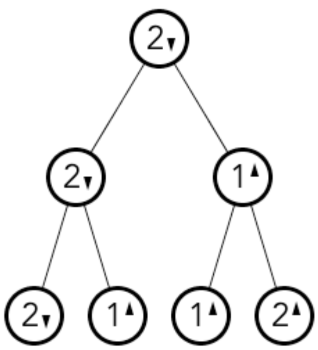


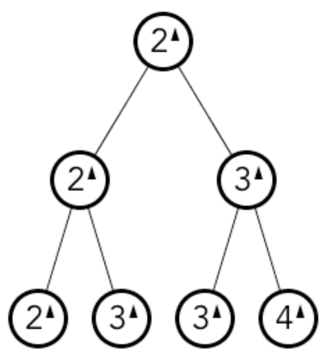
Analysis and Estimate

For the case where the barrel is initially on variation 1, we can map out the possible variations as a binary tree. The total number of variations for this initial configuration are the number of leaves at the bottom of the tree, or 2^{19}. Variation 8, at the top of the staircase, works the same way.



For the case where the barrel is on variation 2, there are two different trees, depending on whether the staircase is ascending or descending. These are also binary trees, which can be calculated as 2^{19}, but they contain one variation in common, in which the aleatoric wheel produces a long succession of zeroes resulting in no change. This same situation may be applied to variations 2 through 7.





Thus my estimate for the number of variations per barrel is

$B = 2 * 2^{19} + 6 * (2 * 2^{19} - 1)$

or

$B = 7*2^{20} - 6$

or

B = 7,340,026

My estimate for the total variations, taking both barrels into account, is

$V = B^2$

or

V = 53,875,981,680,676

How Mahillon's numbers were derived

Mahillon did not provide an explanation of how his calculation was derived, but we can make some educated guesses.

My number is B^2 in which B is the number of variations per barrel, with both barrels producing the same number of variations.

Likewise, Mahillon's number 14,513,461,557,741,527,824 is a perfect square indicating that Mahillon estimated 3,809,653,732 variations per barrel.

This number corresponds to the number of 20-digit long base-8 numbers with adjacent digits differing by one or less (the 20th integer in OEIS A126362).

Mahillon correctly assumed 20 phrases per barrel. His calculation expects to see sequences like 1,2,2,3,2,3,3... This reasoning assumes the staircase gear can shift direction. I am assuming, based on Van Tiggelen's description, that the the staircase gear turns in only one direction, and the variations must continue ascending before they can descend. Once we've done 1,2,3, we must do either 3 or 4 - we can't do 2 again until we ascend up to 8 and then descend back down to 2.

The correct OEIS sequence which takes the staircase effect into account is OEIS A048489.

Years of uniqueness

Mahillon's time estimate of 138 trillion years is also misleading. His estimate is of the amount of time required to play V=138,065,654,088,104 variations at 5 minutes per piece, or

T = V*5/(60*24*365) = roughly 138 trillion years

While the math is correct, given his assumptions, there are a few problems with his assumptions:

1) Mahillon's estimate of the number of variations is far too large, as I've described above.

2) 5 minutes is misleading. A recording of the Componium provided by the MIM reveals that it played with a tempo of about 128 beats per minute. At 80 measures per piece, this provides about 2.5 minutes of music, rather than 5.

3) The Componium is not actually capable of playing all its unique variations without repetition.

Based on my calculations, I provide a few more reasonable estimates below:

a) The amount of time required to play all the unique variations on an imaginary machine actually capable of playing them in sequence without repetition (Mahillon's number). Based on my new estimate of 53875981680676 variations, I put this number at 512,518,851 years. If we half the piece duration to a more realistic 2.5 minutes, the number is reduced to 256,259,425 .

b) The number of variations one would need to play on a well-maintained Componium before you can expect to hear a single repeat. This is similar to the well known birthday problem. I estimate this as sqrt(2*(53875981680676) * log(1.0/(1-0.5))) = roughly 8,642,220 iterations, which makes for 41 years, given a 2.5 minute piece.

c) The number of variations one would need to play before one can expect to have heard each one at least once. This is essentially the same as the coupon collector's problem. I put this number of iterations at V*H(V), where H(V) is a harmonic number, or 1,734,533,003,667,643, which makes over 8 billion years, given a 2.5 minute piece.†

Since the Componium has not remained in playable condition for more than a few decades, it seems unlikely that any of these numbers can properly be tested on the actual device. We can only hope that the machine, unplayable since the mid 1960s, will once again be restored, and live on to entertain and fascinate the public in the distant future.

Jim Bumgardner, 12/8/2013

Thanks to Alex Boxer for spotting the connection between Mahillon's number and OEIS A126362, and Craig Kaplan for continual advice and encouragement.

References

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[6] Bass, Thomas A. : *The Eudomonic Pie*, Houghton Mifflin, 1985

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† My estimates b and c assumes that the aleatoric clockwork in the Componium behaves like a truly random number source, hence my use of the qualifier "well maintained". In fact, it does not. Stan Sek showed that under the conditions in which he tested it, it is highly non-random in character [5]. Van Tiggelen notes that Sek's measurements were likely influenced by the the imperfect condition of the Componium at the time of measurement, however it should be noted that imperfect roulette wheels, are more the rule rather than the exception, a fact which has led to cheating in casinos via concealed computers. [6] This non-random character greatly reduces the number of consecutive runs of "heads" or "tails". Also, the 14-step cycle of the staircase wheel increases the presence of variations 2-7 at the expense of 1 and 8. These non-flat characteristics greatly increase the likelihood that repeats will be played by the machine. 41 years is too high, and 8 billion years is too low.